**Reasoning and Proof** **Name:_______KEY_____________**
**U1 Cumulative Review**

**Lesson 1:**
Know how to: Identify the hypothesis and conclusion of a claim – Write conjectures in if-then form – Determine whether conclusions can be drawn from certain known facts and given information – Understand the difference between, and provide examples of deductive and inductive arguments

**Lesson 2:**
Know how to: Use counter examples to show that an algebraic expression is not true – Use deductive reasoning to show that an algebraic expression is true

**1.** The following statement is true:

If a student at Western Michigan University is
on the volleyball team, then the student is at least 5'7"

**a.** Identify the hypothesis and conclusion of the statement

*Hypothesis- If a student at Western Michigan University is on the volleyball team*
*Conclusion- then the student is at least 5'7"*

**b.** Write the converse of the statement. Is the converse of the statement necessarily true?

*If a student at Western Michigan University is at least 5'7", then the student is on the v-ball team.*

**c.** If Samantha is on the WMU volleyball team, what conclusion, if any, can be made?

*Then is at least 5'7".*

**d.** If Jaclyn, a student at WMU, is 5'9", what conclusion, if any, can be made?

*One cannot be made.*

**2.** Rewrite each of the following statements in if…then form

**a.** All seniors at Black River must complete a senior project to graduate.

*If a student at Black River is a senior, then the student must complete a senior project to graduate.*

**b.** The product of two negative numbers is always a positive number.

*If you multiply two negative numbers, then its product is always a positive number.*

**3.** Use Inductive reasoning to explore the result of the following procedures. Then use deductive reasoning to prove that the pattern you observe will always work.

| |
|---|
| Pick a number.<br>Double it.<br>Add 3 to the result.<br>Multiply that result by 5.<br>Subtract 7 from that result. |

| *<u>Inductive:</u>* | *<u>Deductive:</u>* | *<u>Check:</u>* |
|---|---|---|
| Picked the number 5. | *5(2x+3)-7* | *10(5)+8=58?* |
| *5 x 2=10* | *10x+15-7* | *50+8=58* |
| *10+3=13* | *10x+8* | |
| *13x5=65* | | |
| *65-7=58* | | |

**4.** Prove or disprove each of the following statements

**a.** The sum of two odd numbers is an odd number.

*(2n+1)+(2n+1)=4n+2*
*4n+2 is always even because it is divisible by 2 (4n+2=2(n+1)). Thus, this statement is* ***false.***

**b.** The product of two odd numbers is an odd number.

*(2n+1)(2m+1)=4mn+2n+2n+1*
*=2(2mn+n+m)+1*
*2(2mn+n+m)+1 is an odd number. Thus, this statement is* ***true.***

**c.** The sum of three consecutive numbers is divisible by 3.

*n+(n+1)+(n+2)=3n+3*

*3n+3 is divisible by 3 since 3n+3=3(n+1). Thus, this statement is* ***true****.*

**Inequalities and Linear Programming**

**Name:______KEY______**

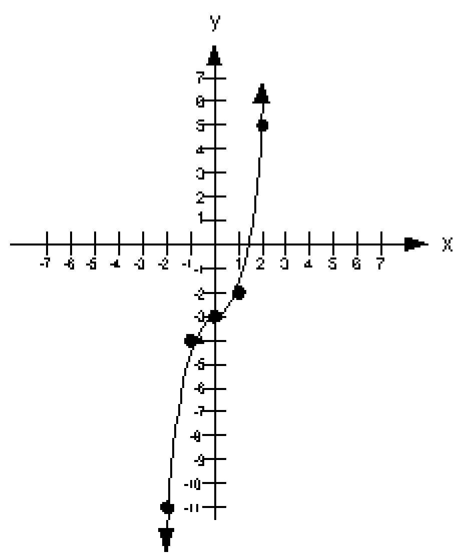
**___**

**Unit 2 Cumulative Review**

**Lesson 1:**
Know how to: Solve an inequality given the graph of a function – solve quadratic inequalities using sketches of the graph and algebraic reasoning – solve complex inequalities using sketches of the graph and algebraic reasoning. Represent solutions to inequalities symbolically – with a number line graph – and with interval notation.

**Lesson 2:**
Know how to: Graph and solve systems of linear inequalities – identify and write inequalities to model linear programming problems – identify and write objective functions of linear programming problems – use graphs to solve linear programming problems.

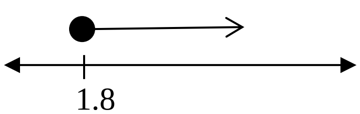
**1.** Use information from this graph of the function $f(x)$ to answer the following questions.




**a.** Find $f(x) = 3$
1.8

**b.** Describe the values of $x$ for which $f(x) \geq 3$ using words, symbols, a number line graph, and interval notation.
When x is greater than or equal to 1.8.

$$x \geq 1.8$$

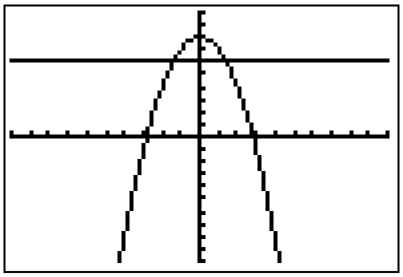



**c.** Find $f(-2)$
-11

**2.** For each inequality below:

- Make a sketch to show how the functions and constants in the inequality are related
- Use algebraic reasoning to locate the key intercepts and points of intersection
- Combine what you learn from your sketch and algebraic reasoning to solve the inequality
- Describe each solution set using symbols, a number line graph, and interval notation

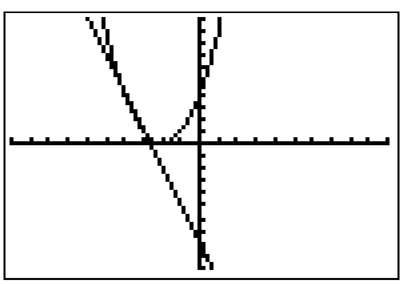
**a.** $8 - x^2 \leq 6$



Algebra:
$8 - x^2 = 6$
$-x^2 = -2$
$x^2 = 2$
$x = \sqrt{2}$
$x \approx 1.414$

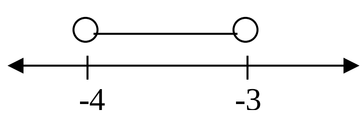
Sketch to solve:
$x \leq -1.414$ or $x \geq 1.414$
$(-\infty, -1.414] \cup [1.414, \infty)$


**b.** $x^2 + 4x + 4 < -3x - 8$

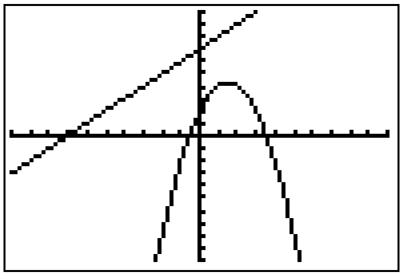


Algebra:
$x^2 + 4x + 4 = -3x - 8$
$x^2 + 4x + 12 = -3x$
$x^2 + 12 + 7x = 0$
$(x + 4)(x + 3) = 0$

Sketch to solve:
$-4 < x < -3$
$(-4, -3)$

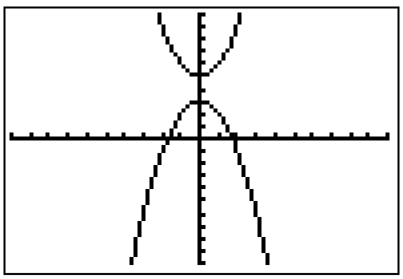


c. $7 + x > 2 + 3x - x^2$



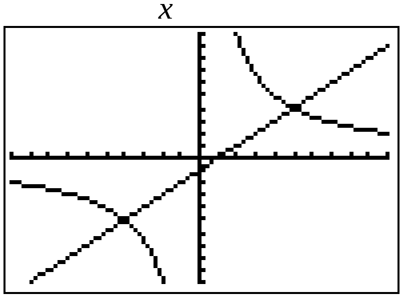
*They do not intersect. All real solutions or all $x$ values are solutions to this inequality.*

**d.** $3 - x^2 \geq x^2 + 5$



*They do not intersect. There are no solutions to this inequality.*

**e.** $x - 1 < \dfrac{20}{x}$



Algebra:

$x - 1 = \dfrac{20}{x}$

$x^2 - x = 20$

$x^2 - x - 20 = 0$

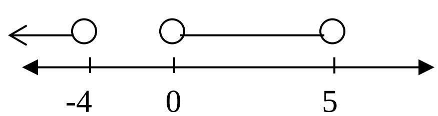
$(x - 5)(x + 4) = 0$

$x = 5$ or $x = -4$

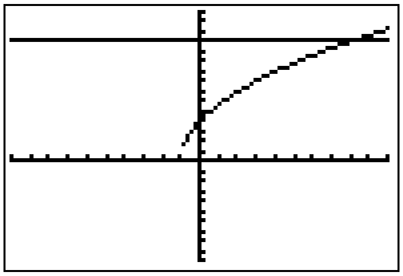
Sketch:

$0 < x < 5$ or $x < -4$

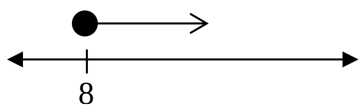
$(0,5) \cup (-\infty, -4)$



**f.** $4\sqrt{x+1} \geq 12$

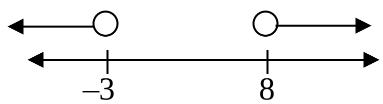


Algebra:
$4\sqrt{x+1} = 12$
$\sqrt{x+1} = 3$
$x + 1 = 9$
$x = 8$
Using Sketch:
$x \geq 8$
$[8, \infty)$

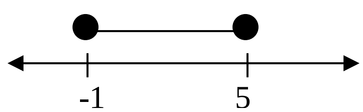



**3.** Describe each of the following solution sets using symbols, a number line graph, and interval notation. (One of the three is provided for you.)

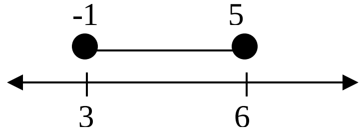
**a.** $x < -3$ or $x > 8$
$(-\infty, -3) \cup (8, \infty)$



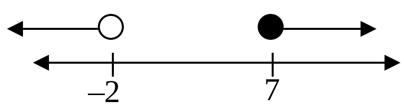

**b.** $x \geq -1$ and $x \leq 5$
$[-1, 5]$



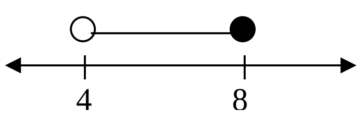

**c.** [3, 6]
$3 \leq x \leq 6$




**d.** $(-\infty, -2) \cup [7, \infty)$
$x < -2$ or $x \geq 7$

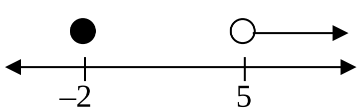



**e.**




$4 < x \leq 8$
$(4, 8]$

**f.**




$x = -2$ or $x > 5$ $(5, \infty)$*****note that -2 is not an interval so has no notation

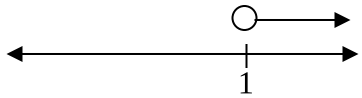
**4.** Use algebraic reasoning to solve the following inequalities. Represent the solutions using symbols and a number line graph.

**a.** $4x - 3 > -7x + 8$

$11x - 3 > 8$

$11x > 11$

$x > 1$



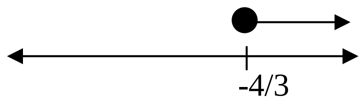

**b.** $2x + 4 \leq 8x + 12$

$2x \leq 8x + 8$

$-6x \leq 8$

$x \geq -\frac{8}{6}$

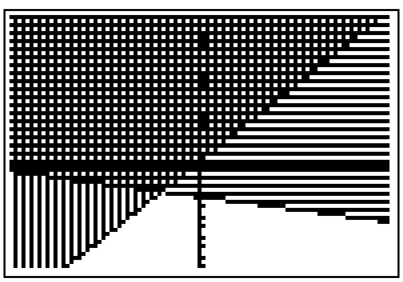
$x \geq -\frac{4}{3}$




**5.** Graph solutions to each of the following systems of linear equations. Remember to use solid lines for solutions that include the line and dashed lines for points on the boundary that are excluded from the solution.

**a.** $3x - 2y \leq -1$

$x + 4y \geq -12$

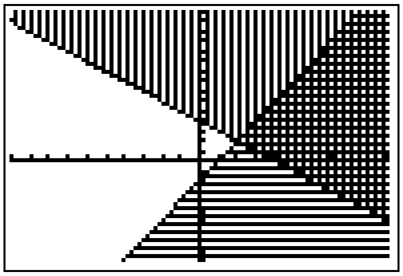


*Solve each equation for y and plug into calculator:*

$y \geq 1.5x + .5$

$y \geq -.25x - 3$

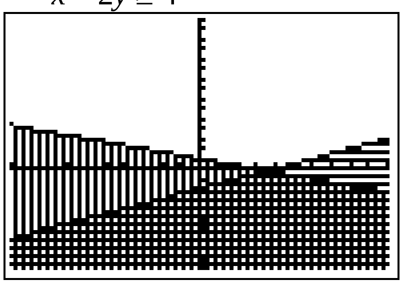
**b.** $x + y \geq 4$
$2x - y > 2$



*Solve each equation for y and plug into your calculator:*

$y \geq -x + 4$
$y < 2x - 2$

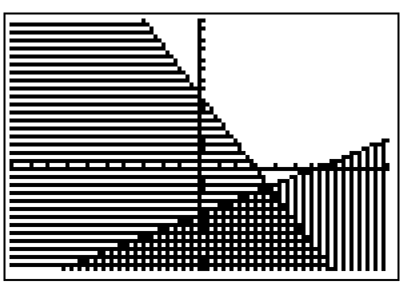
**c.** $x + 3y < 3$
$x - 2y \geq 4$



*Solve each equation for y and plug into your calculator:*

$y < -\frac{1}{3}x + 1$
$y \leq .5x - 2$

**d.** $3x - 4y > 18$
$5x + 2y \leq 15$



*Solve each equation for y and plug into your calculator:*

$y < .75x - \frac{18}{4}$
$y \leq -\frac{5}{2}x + \frac{15}{2}$

**6.** The drama club is selling tickets to its play. An adult tickets costs \$15 and a student ticket costs \$11. The auditorium will seat 300 ticket holders. The drama club wants to collect at least \$3,360 from ticket sales. Write and graph a system of inequalities that describes how many of each type of ticket the club must sell to meet its goal.

$15a + 11s \geq 3{,}360$
$a + s \leq 300$

**7.** The Dutch Flower Bulb Company bags a variety of mixtures of bulbs. There are two customer favorites – the Moonbeam and Sunshine mixtures – each containing a mixture of daffodils and jonquils.

- The Moonbeam mixture contains 30 daffodils and 50 jonquils
- The Sunshine mixture contains 60 daffodils and 20 jonquils
- The company imports 300,000 daffodils and 260,000 jonquils for sale each year
- The profit for each bag of Moonbeam mixture is \$2.30, while the profit for each bag of Sunshine mixture is \$2.50

**a.** Write inequalities and an objective function to help decide how many bags of each mixture the company should make in order to maximize profit without exceeding available supplies.

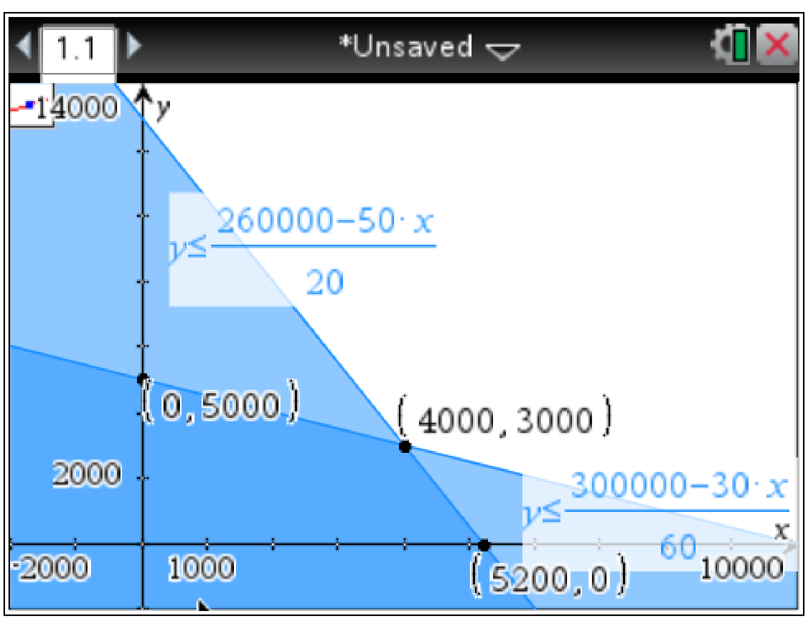
*Variables: x* is moonbeam mixes, *y* is Sunshine mixes

*Daffodils:* $300{,}000 \geq 30x + 60y$

*Jonquils:* $260{,}000 \geq 50x + 20y$

*Profit:* $P = 2.3x + 2.5y$

**b.** Use graphs to solve this linear programming problem.




Check the corners:
(0, 5000) → \$12,500
(4000, 3000) → \$16,700
(5,200, 0) → \$11,960
The best option if 4000 moonbeams, and 3000sunbeams, for profit of \$16,700

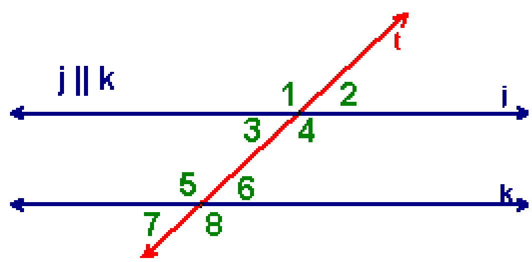
**Similarity and Congruence** **Name:______________________**
**Geometric Reasoning and Proof**
**Unit 3 Cumulative Review**

**Lesson 1:**
Know how to: Identify special pairs of angles of intersecting lines – Identify special pairs of angles of parallel lines – Solve problems related to angles of intersecting lines and parallel lines

**Lesson 2:**
Know how to: Recognize similar triangles – Determine which similarity statement applies to similar triangles – Find the scale factor and side lengths of similar triangles – Use similar triangles to solve problems

**Lesson 3:**
Know how to: Recognize congruent triangles – Determine which congruency statement applies to congruent triangles – Use congruent triangles to prove properties of triangles and special quadrilaterals

**1.** In the diagram below $j \parallel k$




**a.** Which line is the transversal?
*t*

**b.** Name a pair of vertical angles and state their relationship to each other.
*angles 2 and 3, they are congruent*

**c.** Name all pairs of alternate interior angles and state their relationship to each other.
*angles 3 and 6, and 5 and 4 are alternate interior angles, they are congruent to each other*

**d.** Name a pair of corresponding angles and state their relationship to each other.

*Angles 1 and 5, they are congruent*

**e.** Name a pair of alternate exterior angles and state their relationship to each other.

*Angles 2 and 7 are congruent*

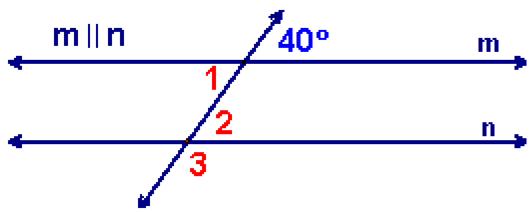
**f.** Name a pair of interior angles on the same side of the transversal and state their relationship to each other.

*Angles 3 and 5, they add up to 180*

**f.** Name a pair of exterior angles on the same side of the transversal and state their relationship to each other.

*Angles 1 and 7, they add up to 180*

**2.** In the diagram below *m* || *n*.




**a.** Find m∠1

40° *by vertical angles*

**b.** Find m∠2

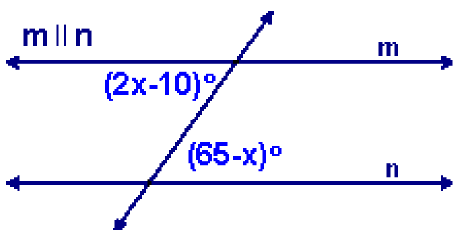
40° *by alternate interior angles*

**c.** Find m∠3

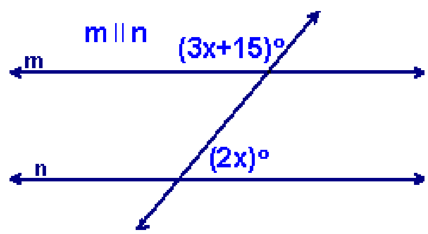
140° *because angle 2 and 3*=180°

**3.** In each of the diagrams below *m* || *n*. Find the measure of *x*.

**a.**



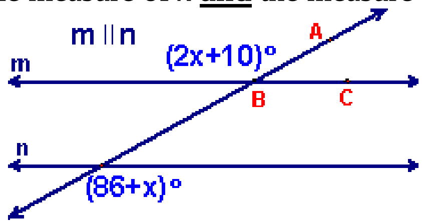

**b.**




$(2x - 10) = (65 - x)$ vertical angles
$2x = 75 - x$
$3x = 75$
$x = 25$

$(3x + 15) + 2x = 180$ by vertical angles and linear pair
$5x + 15 = 180$
$5x = 165$
$x = 33$

**4.** In the diagram $m \parallel n$. Find the measure of $x$ **and** the measure of $\angle ABC$.




$(2x + 10) = (86 + x)$ by alternate interior angles

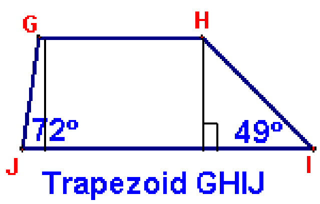
$2x = 76 + x$

$x = 76$

$2(76) + 10 = 162$

$180 - 162 = 18° = m\angle ABC$

**5.** Quadrilateral GHIJ is a trapezoid. Find m∠G **and** m∠H.




$90 + 49 + x = 180$

$x = 41$

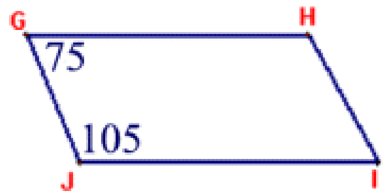
$90 + y + 72 = 180$

$y = 18$

Thus, $\angle H = 90 + 41 = 131$

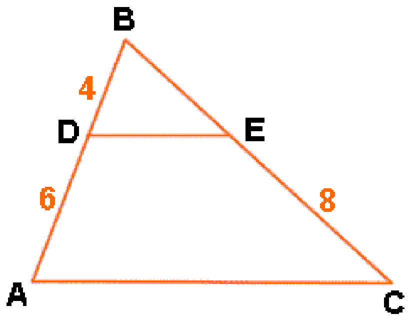
and $\angle G = 18 + 90 = 108.$

**6.** In quadrilateral GHIJ, m∠G = 75° and m∠J = 105°. Is GH || JI? Explain.




Extend line GJ and add point K onto it. Then the angle created ($\angle HGK$) must equal 105 degrees because it froms a linear pair with $\angle HGJ$. Thus, the corresponding angles $\angle HGK$ and $\angle IJG$ are equal. A similar argument can be made for the other angles. Then, by the Parallel lines postulate GH || JI.

**7.** In the diagram DE || AC, BD = 4, DA = 6, and EC = 8. Find BC.

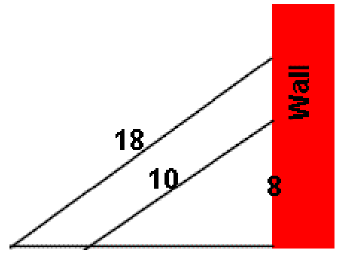



$\frac{4}{10} = \frac{2}{5}$

$\frac{2}{5} = \frac{x}{x+8}$

$5x = 2x + 16$
$3x = 16$
$x = 5\frac{1}{3}$
$BC = 5\frac{1}{3} + 8 = 13\frac{1}{3}$

**8.** Two ladders are leaned against a wall such that they make the same angle with the ground. The 10' ladder reaches 8' up the wall. How much further up the wall does the 18' ladder reach?



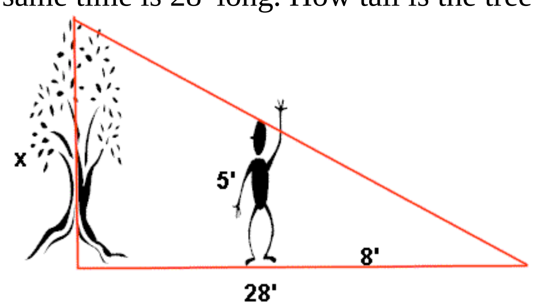

$\frac{10}{18} = \frac{5}{9}$

$\frac{5}{9} = \frac{8}{x}$

$5x = 72$
$x = 14.4$

$14.4 - 8 = 6.4$

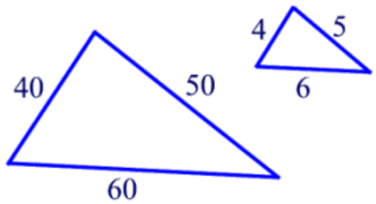
**9.** At a certain time of the day, the shadow of a 5' boy is 8' long. The shadow of a tree at this same time is 28' long. How tall is the tree?




$\frac{8}{28} = \frac{2}{7}$
$\frac{2}{7} = \frac{5}{x}$
$2x = 35$
$x = 17.5$

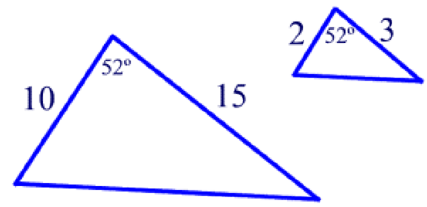
**11.** For each of the following pairs of triangles, determine whether they are similar. If so, state the similarity condition that proves they are similar. If not, explain why not.

**a.**



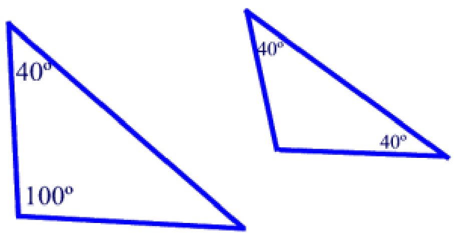

*They are similar by SSS. Ratio is 10.*

**b.**




*They are similar by SAS. Ratio is 5.*

**c.**



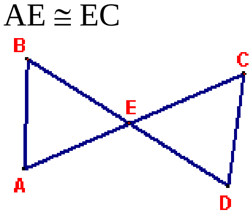

*They are similar by AA.*

**d.** Provide an example of two triangles that are not similar and explain why not.

*Have fun!!*

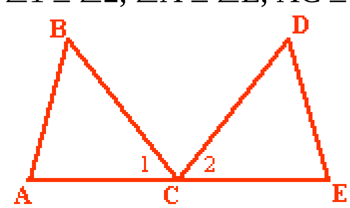
**12.** For each of the following pairs of triangles determine whether the given information can be used to prove the triangles congruent. If so, state the congruency theorem, if not, explain why not.

**a.** E is the midpoint of BD, AE ≅ EC



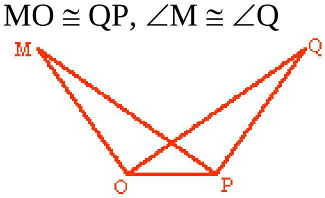

*Similar by SAS.*

**b.** ∠1 ≅ ∠2, ∠A ≅ ∠E, AC ≅ EC



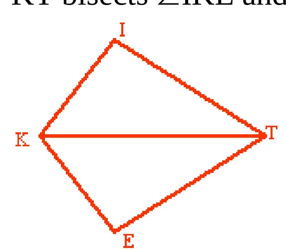

*Similar by ASA.*

**c.** MO ≅ QP, ∠M ≅ ∠Q



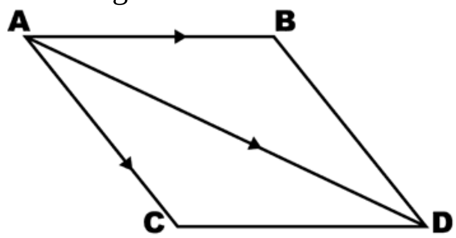

*Not enough information.*

**d.** KT bisects ∠IKE and ∠ITE



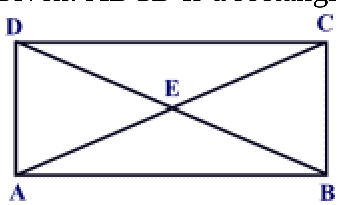

*Similar by ASA.*

**13.** Prove that the diagonals of a parallelogram divide the parallelogram into two congruent triangles.



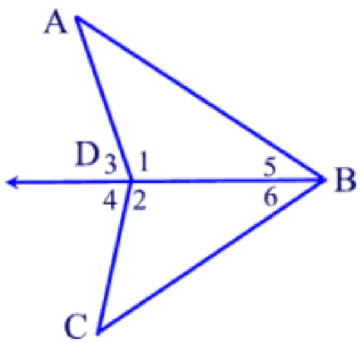

$AC = DB$ by definition
$\angle C = \angle B$ by definition
$AB = DC$ by definition
Thus, by SAS $\triangle ADC \cong \triangle DAB$.

**14.** Given: ABCD is a rectangle; Prove: ΔCEB is isosceles




$DB$ bisects $AC$ at $E$ by definition
$CA$ bisects $DB$ at $E$ by definition
$DE = EB$, $AE = EC$, $DB = AC$ by definition
$DB = DE + EB$ from drawing
$AC = AE + EC$ from drawing
$DE + EB = AE + EC$ by substitution
$2EB = 2EC$ by substitution
$EB = EC$ by algebra
Thus, $\triangle CEB$ is isosceles.

**15.** Given: AD ≅ CD and ∠3 ≅ ∠4; Prove: DB bisects ∠ABC




$\angle 3 = \angle 4$ given
$\angle 3 + \angle 1 = 180$ linear pair
$\angle 4 + \angle 2 = 180$ linear pair
$\angle 3 + \angle 1 = \angle 4 + \angle 2$ substitution
$\angle 1 = \angle 2$ substitution
$DB = DB$ same line
$\triangle ADB \cong \triangle CDB$ by SAS
$\angle 5 = \angle 6$ by CPCTC
Thus, DB bisects ∠ABC

**16.** State whether the following are true or false, and, if false, give a counterexample.

a. If a quadrilateral has one pair of opposite sides parallel and congruent, then the quadrilateral is a parallelogram.

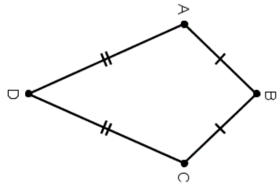
*True see quadrilateral cases for full proof*

b. If a quadrilateral has a diagonal that divides it into two congruent triangles, then the quadrilateral is a parallelogram.

*True see quadrilateral cases for full proof*

c. If a quadrilateral has two distinct pairs of consecutive sides congruent, then the quadrilateral is a parallelogram.

*False*



d. If a quadrilateral has diagonals that bisect each other, then the quadrilateral is a parallelogram.

*True*

e. If a quadrilateral has four congruent angles, then the quadrilateral is a parallelogram.

*True...in fact this is a rectangle and must therefore be a parallelogram*

f. If a quadrilateral has two pair of sides that are parallel, then the quadrilateral is a parallelogram.

*True...definition of parallelogram*

**17.** If a quadrilateral has 2 pair of opposite sides congruent and parallel, what is the *most specific* conclusion you can make?

a. The quadrilateral is a square
b. The quadrilateral is a rectangle
c. The quadrilateral is a parallelogram
d. The quadrilateral is a rhombus

*C*

**18. Study your Quadrilaterals Precedents sheet.**

a. A quadrilateral is a parallelogram if and only if each diagonal divides the quadrilateral into ___*two congruent*_____________________ triangles.

b. A quadrilateral is a parallelogram if and only if the diagonals ___*bisect*___ each other.

c. A quadrilateral is a rectangle if and only if the diagonals are *congruent* and _________*bisect*___________ each other.

d. A quadrilateral is a rhombus if and only if the diagonals are ___*perpendicular*____ bisectors of each other.

e. A quadrilateral is a square if and only if the diagonals are *congruent* and *perpendicular*___ bisectors of each other.

f. A quadrilateral is a kite if and only if *one diagonal* is the perpendicular bisector of the other.

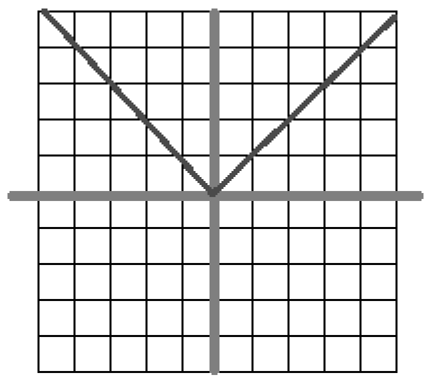
g. It you connect the midpoints of consecutive sides of ANY quadrilateral in order, then the resulting shape is charged with being a parallelogram. This is known as the ___*Midpoint*___ Connector Theorem for _*Quadrilaterals*______.

**Integrated Advanced Algebra/Geometry**
**Name____________________**
**Unit 8 Lesson 1 Form C**

1. On the grid below, draw a graph of a function that does not have an inverse. Then explain why your function does not have an inverse.

*Any graph that has repeated y-values will not have an inverse. It will not pass the horizontal line test, the horizontal line will pass through more than one point on the graph.*

$f(x) = |x|$



2. Find an algebraic rule for the inverse of each function.
   a. $f(x) = 7x + 4$
   (switch f(x) for a y, swap x and y, solve for y, replace y with inverse notation)

$$y = 7x + 4$$
$$x = 7y + 4$$
$$x - 4 = 7y$$
$$\frac{x-4}{7} = y$$
$$\frac{x-4}{7} = f^{-1}(x)$$

   b. $g(x) = \frac{5x-2}{3}$
   (switch g(x) for a y, swap x and y, solve for y, replace y with inverse notation)

$$y = \frac{5x-2}{3}$$
$$x = \frac{5y-2}{3}$$
$$3x = 5y - 2$$
$$3x + 2 = 5y$$
$$\frac{3x+2}{5} = g^{-1}(x)$$

3. Suppose a message was first coded by assigning numbers to letters as shown below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | B | C | D | E | F | G | H | I | J |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| K | L | M | N | O | P | Q | R | S | T |

| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| U | V | W | X | Y | Z | |

Brad then encrypted a message using the function $f(x) = 2x - 2$.

a. If the received message is given in encrypted form by 24, 8, 8, 38, 52, 14, 16, 24 what was the message that was sent?

*To decode...add 2 then divide by 2*
*13, 5, 5, 20, 27, 8, 9, 13*
MEET HIM

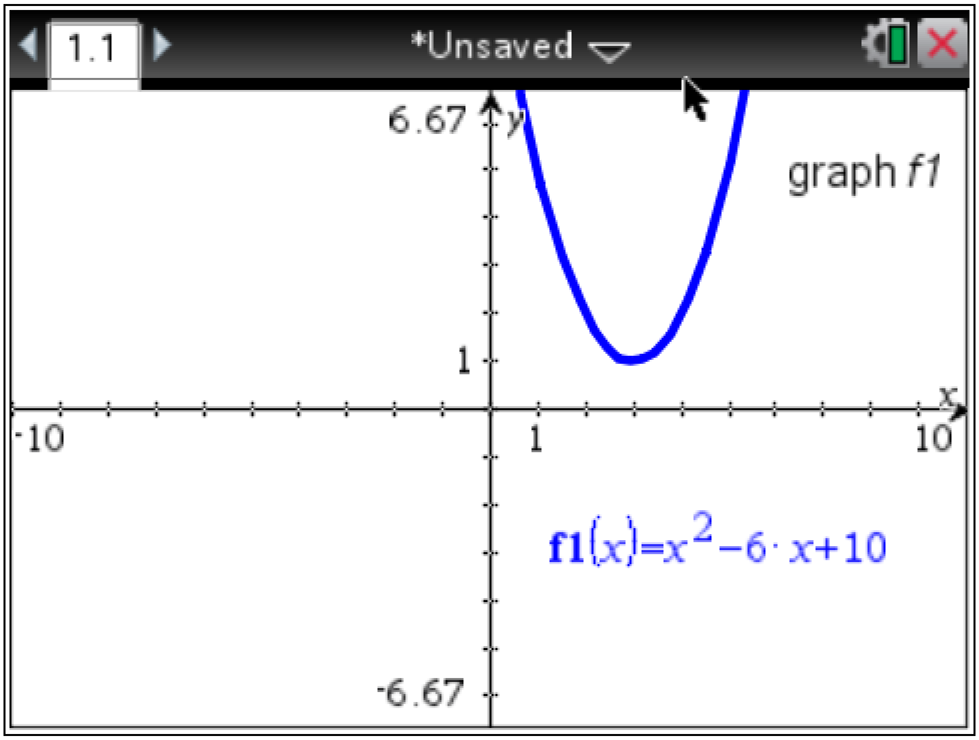
b. Explain in words how you could decode any message encrypted by the function $f(x) = 2x - 2$.

*To decode add 2, then divide by 2*

c. What function would decode any message encrypted by $f(x) = 2x - 2$?

$$f^{-1}(x) = \frac{x+2}{2}$$

d. Sarah wants to use a more complicated function for encrypting messages. She suggests using $g(x) = x^2 - 6x + 10$. What problems might arise if Sarah used this function? Be as specific as possible.




*When you graph the function you can see that the values will be repeated...the codes for 0, 1, and 2 would be the same as 4, 5, 6*

e. Suggest a different quadratic function that would be better for Sarah to use. Explain why your function does not have the same problems as Sarah's function does.

*You just have to use a quadratic that is centered at 0, so 0-27 will not have repeats. Try* $g(x) = x^2 + 1$

The amount of time (in hours) it will take to prepare a mass mailing of a newsletter when there are $x$ people working on the job is determined by the equation $f(x) = \frac{9}{x} + 1$.

f. Find $f(3)$ and explain its meaning in this situation.

$f(3) = \frac{9}{3} + 1 = 4$.
*This means that if 3 people are working it will take 4 hours to complete the mailing*

g. $f^{-1}(1.5) = 18$. Explain what this tells you about the situation.

*Note that is $f^{-1}$, the inverse function. So this means that the inputs are now hours and the outputs are now workers...so to complete the job in 1.5 hours you would need to have 18 people working*

h. Find a formula for $f^{-1}(x)$.

$$y = \frac{9}{x} + 1$$

$$x = \frac{9}{y} + 1$$

$$x - 1 = \frac{9}{y}$$

$$y = \frac{9}{x-1}$$

$$f^{-1}(x) = \frac{9}{x-1}$$

**Integrated Advanced Algebra/Geometry** **Name______-KEY____________**
**Unit 8 Lesson 1 Form C**

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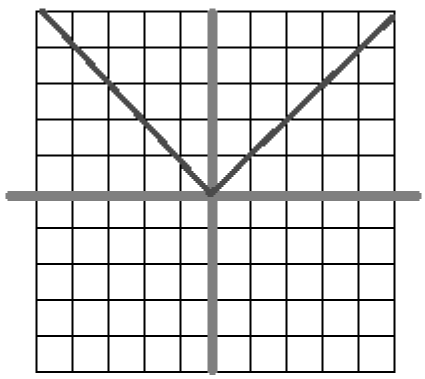
f(x) = |x|



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|---|---|---|---|---|---|---|---|---|---|
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|---|---|---|---|---|---|---|---|---|---|
| K | L | M | N | O | P | Q | R | S | T |

| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
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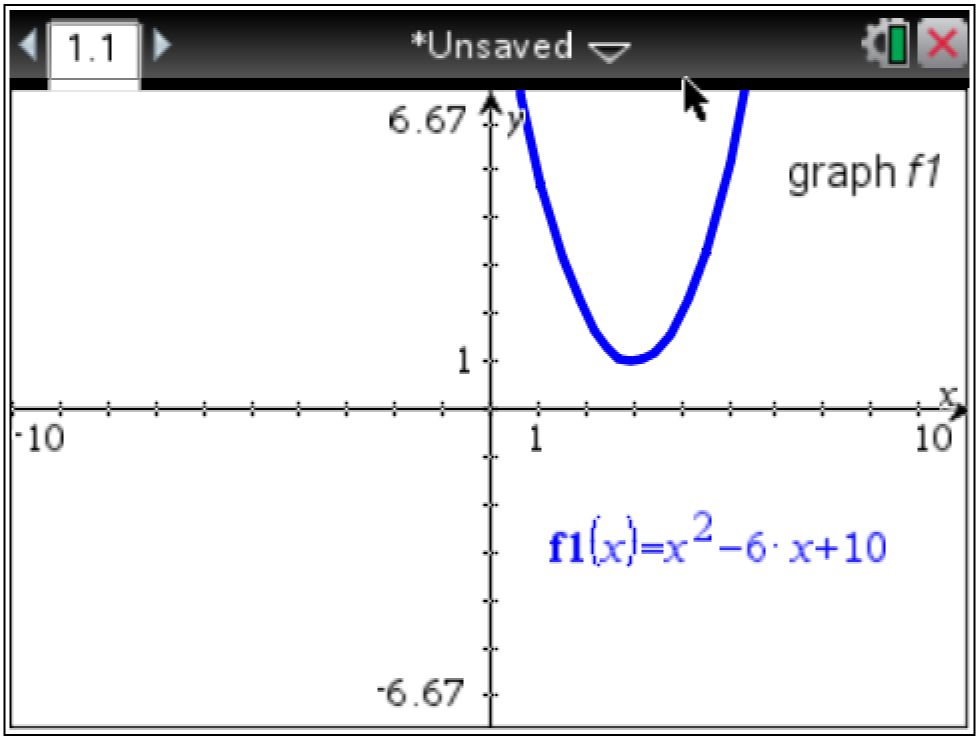


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h. Find a formula for $f^{-1}(x)$.

$$y = \frac{9}{x} + 1$$
$$x = \frac{9}{y} + 1$$
$$x - 1 = \frac{9}{y}$$
$$y = \frac{9}{x - 1}$$
$$f^{-1}(x) = \frac{9}{x - 1}$$

Int. Adv. Alg/Geo
Lesson 8.2 Exam Review                                        Name: _______KEY_______________

**No Calculator Section:**

1. Use your understanding of common logarithms to help complete the following tasks WITHOUT A CALCULATOR.

Find these common (base 10) logarithms *without* using technology.

a. $\log 10^{5.5}$    b. $10^{\log 8}$    c. $\log 1{,}000$    d. $\log .000001$

5.5    8    3    -6

2. Given that $\log 40 \approx 1.6$ and $\log 5 \approx .69$, approximate each of the following expressions using properties of logarithms.

a. $\log 8$    b. $\log 25$    c. $\log 200$    d. $\log \frac{15}{120}$

1.6-.69    .69+.69    1.6+.69    .69-1.6

.91    1.38    2.29    -.91

3. Use what you know about properties of logarithms to answer the following True or False statements. If a statement is False, fix the part of the statement in bold to make the statement true.

_T___ i. $\mathbf{\log \frac{2}{5}} = \log 2 - \log 5$

__T__ ii. $\mathbf{\log}(2x^3) = \log 2 + 3\log x$

__T__ iii. $-3\log m = \mathbf{\log m^{-3}}$

_F___ iv. $2\log 6 = \mathbf{\log 12}$    $2\log 6 = \mathbf{\log 36}$

Int. Adv. Alg/Geo
Lesson 8.2 Exam Review *calculator portion* Name: ______________________

4. Suppose that the decay of a radioactive substance is modeled by the function $h(x) = 80(0.95^x)$, where $x$ is measured in years and $h(x)$ is measured in grams.

a. How much of the substance is there initially?

*80 grams*

b. How much substance is left after 20 years?

*28.6789 grams*

c. How many years will the substance take to decay to 25 grams?

$$25 = 80(.95^x)$$

$$.3215 = (.95^x)$$

$$\log_{.95}.3215 = x \text{ OR } x = \frac{\log .3215}{\log .95}$$

$$x \approx 22.1229 \text{ years}$$

e. What is the half-life of this radioactive substance?

$$40 = 80(.95^x)$$

$$.5 = (.95^x)$$

$$\log_{.95}.5 = x \text{ OR } x = \frac{\log .5}{\log .95}$$

$$x \approx 13.5134 \text{ years}$$

5. Solve each of the following equations algebraically. Show your work.

a. $5(2^{x+1}) = 450$

b. $0.4(5^{3x}) = 12$

$$\mathbf{5(2^{x+1}) = 450}$$

$$\mathbf{2^{x+1} = 90}$$

$$\mathbf{x + 1 = \frac{\log(90)}{\log 2}}$$

$$\mathbf{x = \frac{\log(90)}{\log 2} - 1 \approx 5.49185}$$

$$\mathbf{0.4(5^{3x}) = 12}$$

$$\mathbf{(5^{3x}) = 30}$$

$$\mathbf{3x = \frac{\log(30)}{\log 5}}$$

$$\mathbf{x = \frac{\log(30)}{\log 5} \div 3 \approx .704428}$$

6. The younger you are when you start investing makes an exponentially large difference. The average interest on a high risk stock investment can yield as much as 12% annually. Assume you invest \$1500 in the stock market this year in 2012

a. Write a function $M(t)$ will give the amount of money in the investment $t$ years after opening the account. (This assumes the rate doesn't change and no additional funds are added to the account).

$$M(t) = 1500(1.12^t)$$

b. How much money would be in the account after 10 years?

$$M(t) = 1500(1.12^{10}) \approx 4658.77$$

c. How many years would you need to keep the money in the investment to double your investment?

$$3000 = 1500(1.12^x)$$

$$2 = (1.12^x)$$

$$\log_{1.12} 2 = x \text{ OR } x = \frac{\log 2}{\log 1.12}$$

$$x \approx 6.11626 \; years$$

d. If you could retire on \$2,500,000 , how many years will it before your \$1500 is enough to retire?

$$2{,}500{,}000 = 1500(1.12^x)$$

$$1666.67 = (1.12^x)$$

$$\log_{1.12} 1666.67 = x \text{ OR } x = \frac{\log 1666.67}{\log 1.12}$$

$$x \approx 65.46\ years$$

7. Solve each of the following exponential equations algebraically.

a. $22(0.65^t) = 44$

$$\mathbf{22(0.65^t) = 44}$$

$$\mathbf{0.65^t = 2}$$

$$\boldsymbol{t = \frac{\log(2)}{\log 0.65} \approx -1.60904}$$

b. $6(2^{3x}) = 180$

$$\mathbf{6(2^{3x}) = 180}$$

$$\mathbf{(2^{3x}) = 30}$$

$$\boldsymbol{3x = \frac{\log(30)}{\log 2}}$$

$$\boldsymbol{x = \frac{\log(30)}{\log 2} \div 3 \approx 1.63563}$$

c. $\mathbf{240 = 4(8^t)}$

$$\mathbf{240 = 4(8^t)}$$

$$\mathbf{60 = 8^t}$$

$$\boldsymbol{\frac{\log 60}{\log 8} = t}$$

$$\boldsymbol{1.96896 \approx y}$$

d. $\mathbf{3(5^{2x-1}) - 9 = 18}$

$$\mathbf{3(5^{2x-1}) - 9 = 18}$$

$$\mathbf{3(5^{2x-1}) = 27}$$

$$\mathbf{(5^{2x-1}) = 9}$$

$$\boldsymbol{2x - 1 = \frac{\log 9}{\log 5}}$$

$$\boldsymbol{2x = \frac{\log 9}{\log 5} + 1}$$

$$\boldsymbol{x = (\frac{\log 9}{\log 5} + 1)/2 \approx 1.18261}$$